Sec. 14.4:
Tangent Planes and Linear Approximations

What We Will Go Over In Section 14.4

1. Tangent Lines and Linear Approximations (Calc. I version)
2. Tangent Planes and Linear Approximations (Calc. IIII version)

1. Tangent Planes and Linear Approximations (Calc. I version)

Calc 1 Version…

At what points on its graph is $f(x)$ differentiable?

Graphically…

$f(x)$ is differentiable at a point if when you zoom in to the graph of f at that point, the graph looks like a straight line.

Algebraically…

$f(x)$ is differentiable at $(a, f(a))$ if $\lim_{x \to a} \frac{f(x)-f(a)}{x-a}$ exists.

1. Tangent Planes and Linear Approximations (Calc. I version)

Calc 1 Version…

Linear Approximation…

If $f(x)$ is differentiable at $(a, f(a))$ and $L(x)$ is the equation of the tangent line to f at that point, then f can be approximated by L near $x = a$. (I.e. $f(x) \approx L(x)$ if x is near a).

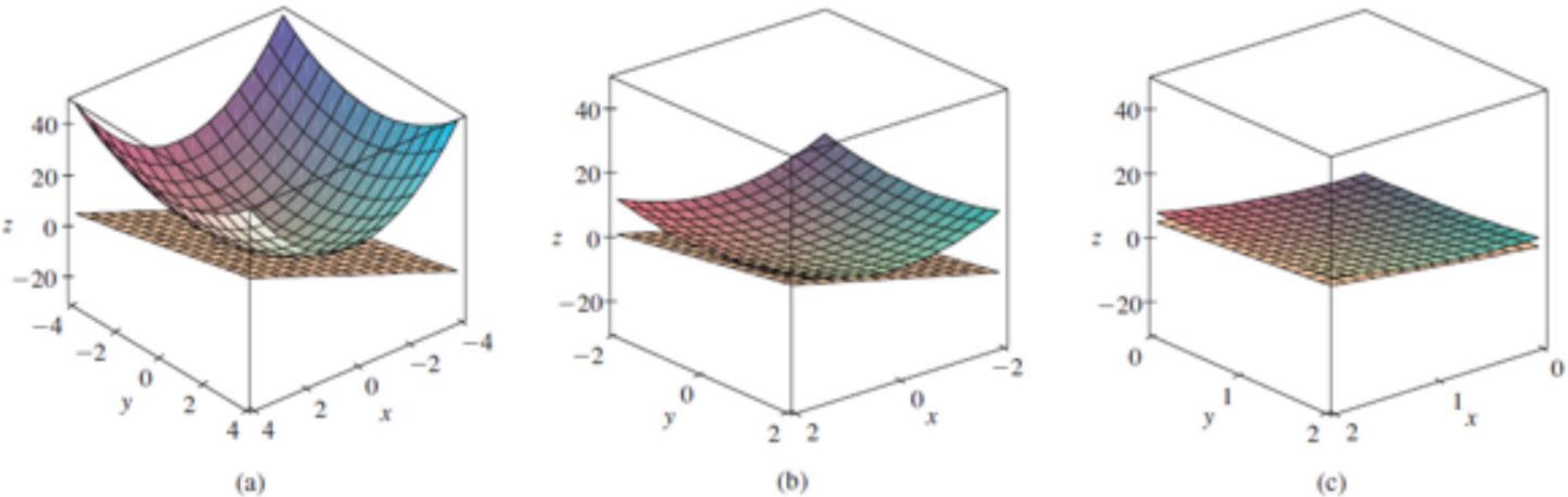
1. Tangent Planes and Linear Approximations (Calc. I version)

Ex 0: Find the linearization to $f(x) = \sqrt{x}$ at $x = 9$, then use the linearization to approximate $\sqrt{10}$.

1. Tangent Planes and Linear Approximations (Calc. III version)

Calc 3 Version…

At what points on its graph is $f(x,y)$ differentiable?



1. Tangent Planes and Linear Approximations (Calc. III version)

Calc 3 Version…

At what points on its graph is $f(x,y)$ differentiable?

Graphically…

$f(x,y)$ is differentiable at a point if when you zoom in to the graph of f at that point, the graph looks like a plane.

Algebraically…

7 Definition

If $z = f(x, y)$, then f is **differentiable** at (a, b) if Δz can be expressed in the form

$$\Delta z = f_x(a, b)\,\Delta x + f_y(a, b)\,\Delta y + \varepsilon_1\,\Delta x + \varepsilon_2\,\Delta y$$

where ε_1 and $\varepsilon_2 \to 0$ as $(\Delta x, \Delta y) \to (0, 0)$.

1. Tangent Planes and Linear Approximations (Calc. III version)

Calc 3 Version…

At what points on its graph is $f(x,y)$ differentiable?

Graphically…

$f(x,y)$ is differentiable at a point if when you zoom in to the graph of f at that point, the graph looks like a plane.

Algebraically…

8 Theorem

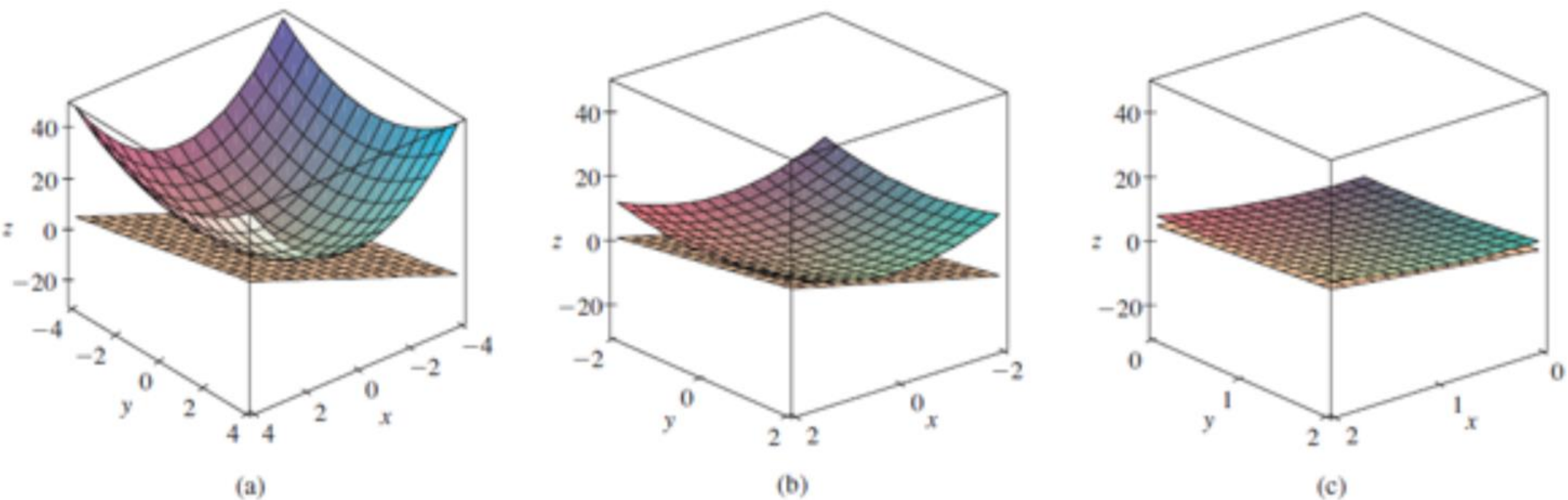
If the partial derivatives f_x and f_y exist near (a, b) and are continuous at (a, b), then f is differentiable at (a, b).

1. Tangent Planes and Linear Approximations (Calc. III version)

Calc 3 Version…

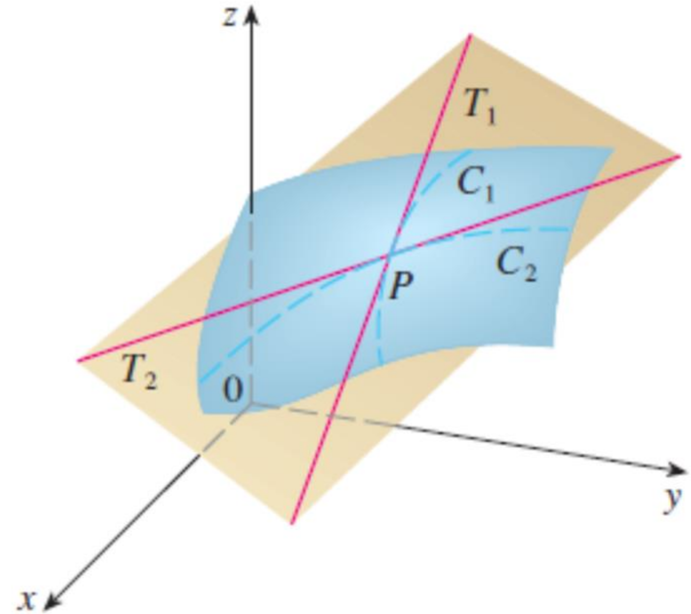
Linear Approximation…

If $f(x,y)$ is differentiable at $(a, b, f(a, b))$ and $L(x, y)$ is the equation of the tangent plane to f at that point, then f can be approximated by L near (a, b). (I.e. $f(x, y) \approx L(x, y)$ when (x, y) is near (a, b)).



1. Tangent Planes and Linear Approximations (Calc. III version)

Derive formula for equation of tangent plane to a 2-variable function



1. Tangent Planes and Linear Approximations (Calc. III version)

Ex 1:

Example 1

Find the tangent plane to the elliptic paraboloid $z = 2x^2 + y^2$ at the point $(1, 1, 3)$.

1. Tangent Planes and Linear Approximations (Calc. III version)

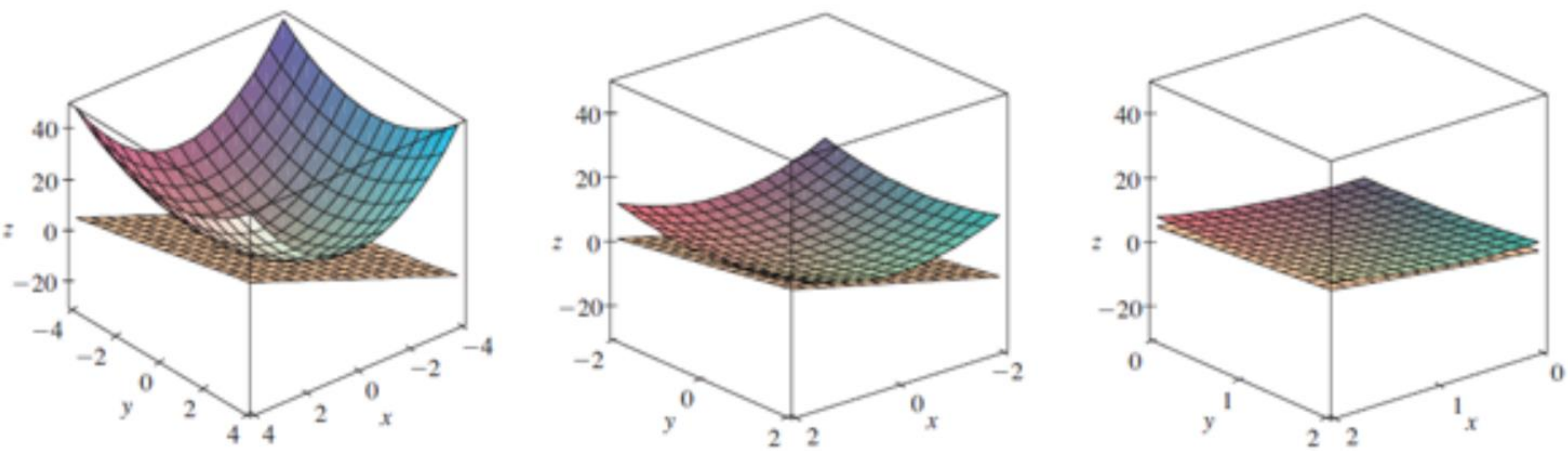
Ex 1:

Example 1

Find the tangent plane to the elliptic paraboloid $z = 2x^2 + y^2$ at the point $(1, 1, 3)$.

Figure 2

The elliptic paraboloid $z = 2x^2 + y^2$ appears to coincide with its tangent plane as we zoom in toward $(1, 1, 3)$.



(a) (b) (c)

1. Tangent Planes and Linear Approximations (Calc. III version)

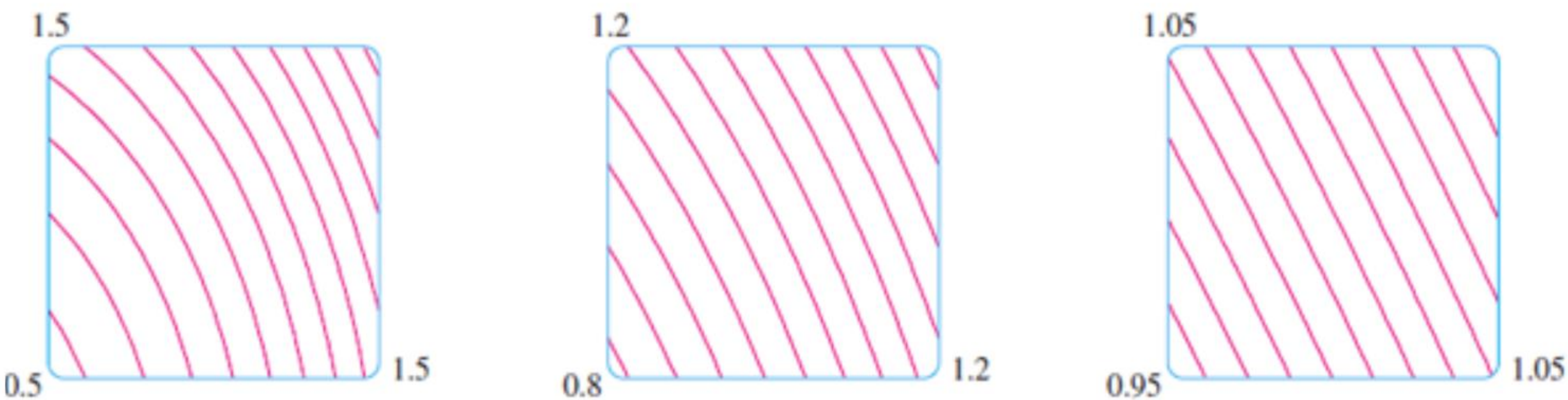
Ex 1:

Example 1

Find the tangent plane to the elliptic paraboloid $z = 2x^2 + y^2$ at the point $(1, 1, 3)$.

Figure 3

Zooming in toward $(1, 1)$ on a contour map of $f(x, y) = 2x^2 + y^2$



1. Tangent Planes and Linear Approximations (Calc. III version)

Ex 2:

Example 2

Show that $f(x, y) = xe^{xy}$ is differentiable at $(1, 0)$ and find its linearization there.

Then use it to approximate $f(1.1, -0.1)$.